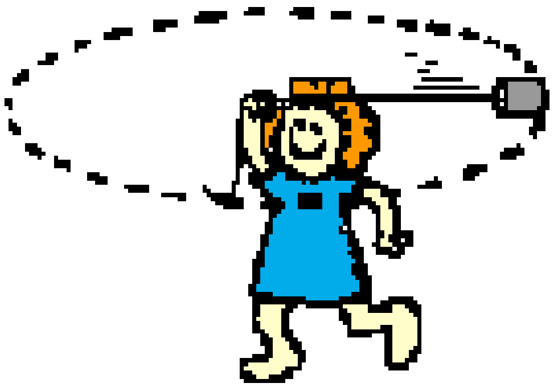
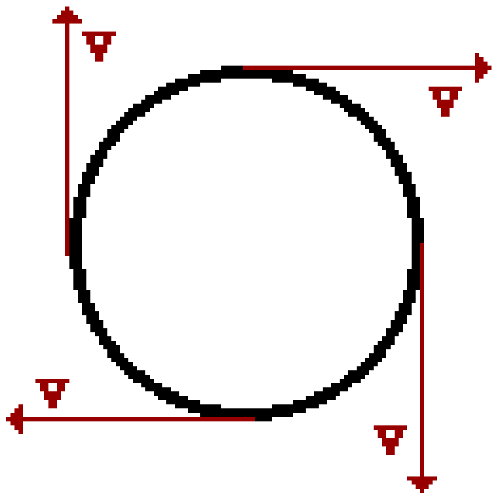
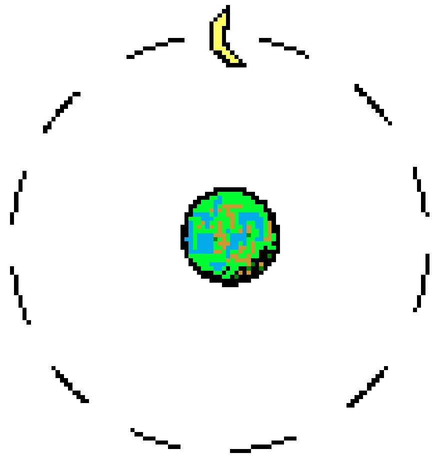
MOTO circolare uniforme

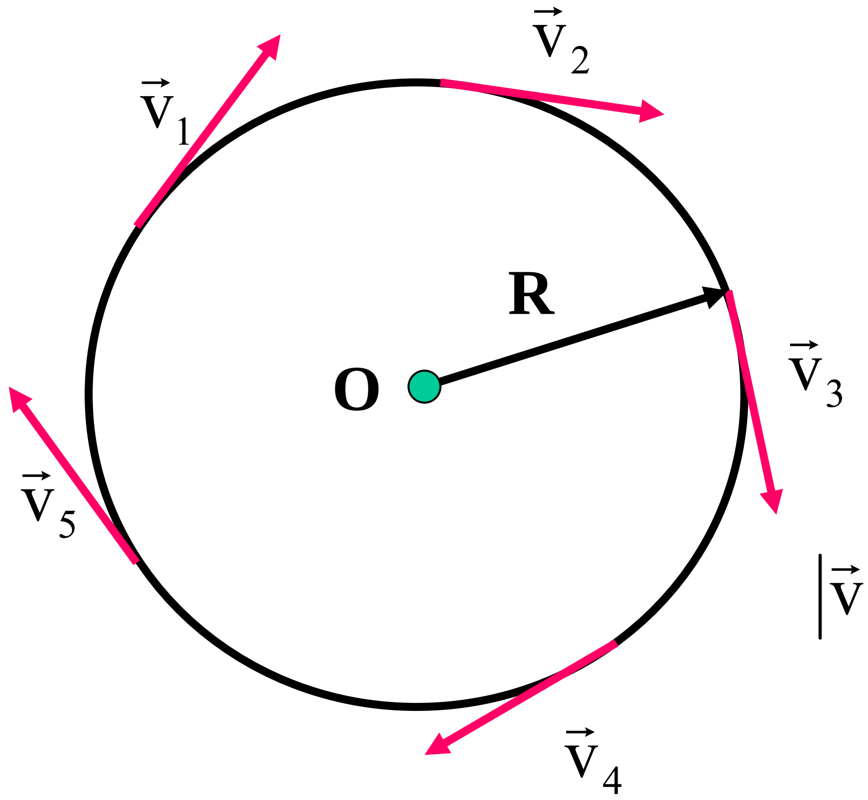






Ad ogni istante la direzione del vettore velocità è tangente alla circonferenza.

velocità



- Modulo costante v
- Direzione variabile
  tangente alla circonferenza

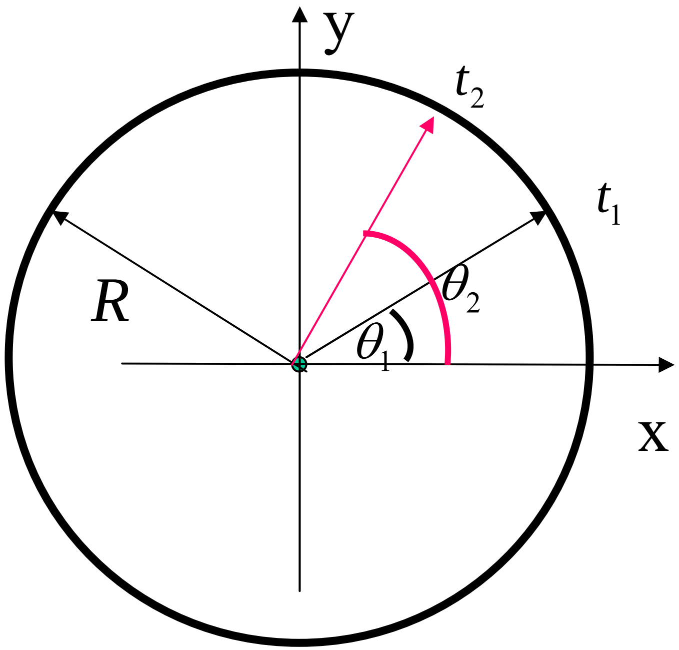
$$|\vec{v}_1| = |\vec{v}_2| = |\vec{v}_3| = |\vec{v}_4| = |\vec{v}_5| = v$$

periodo T

frequenza $f = \frac{1}{T}$ ($f \leftarrow \nu$)

$$v = \frac{2\pi R}{T}$$

velocità angolare



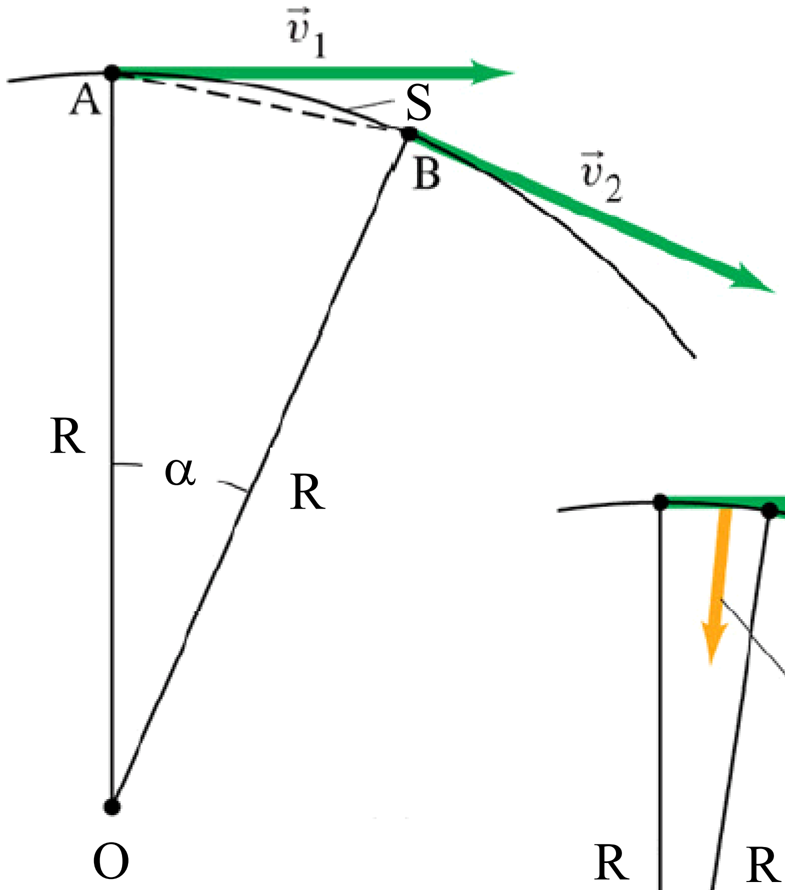
$$\omega_m = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{\mathrm{T}}$$

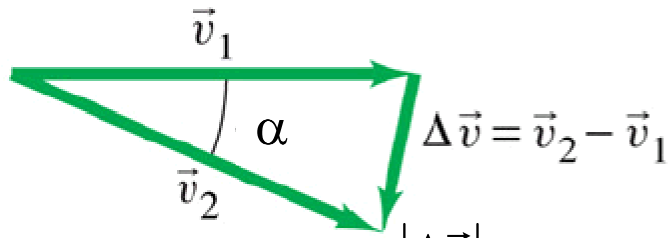
$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

$$T = \frac{2\pi}{\omega} \qquad \mathrm{f} = \frac{1}{\mathrm{T}} = \frac{\omega}{2\pi}$$

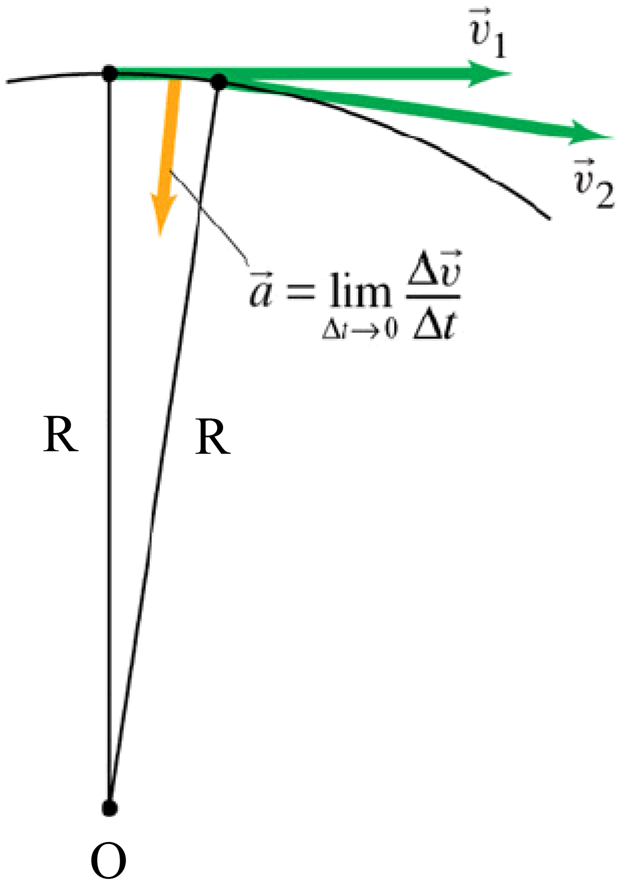
$$\mathrm{v} = \frac{\mathrm{ds}}{\mathrm{dt}} = \frac{\mathrm{d}}{\mathrm{dt}}\mathrm{R}\theta = \mathrm{R}\frac{\mathrm{d}\theta}{\mathrm{dt}} = \omega\mathrm{R}$$

accelerazione



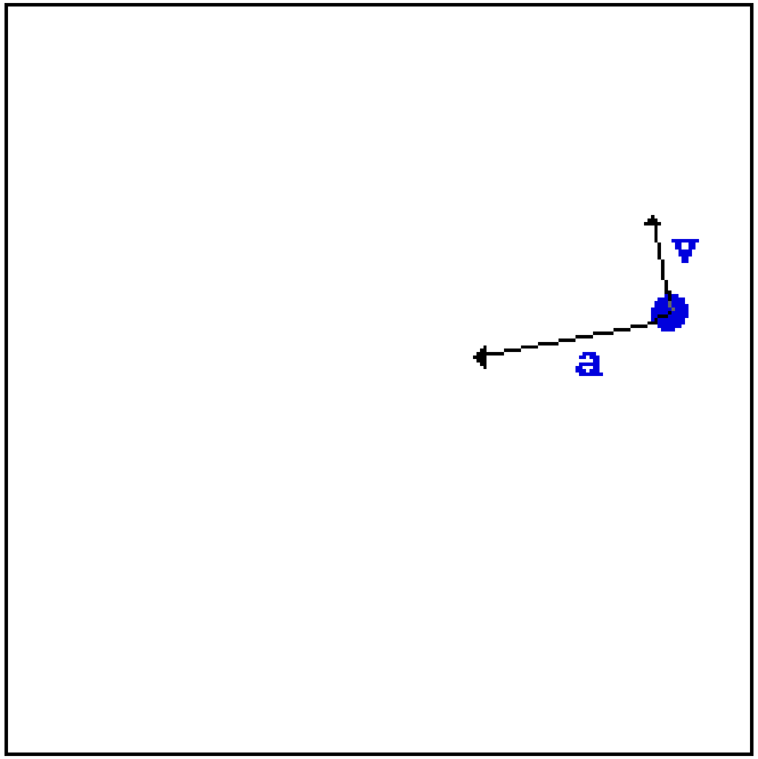


$$\left|\Delta\vec{\mathrm{v}}\right| \cong \mathrm{v}\alpha = \mathrm{v}\omega\Delta \mathrm{t}$$



$$\left|\vec{a}\right| = \frac{\left|\Delta\vec{\mathrm{v}}\right|}{\Delta t} \cong \mathrm{v}\omega = \omega^2 R = \frac{\mathrm{v}^2}{R}$$

Velocità e accelerazione



$$T = \frac{2\pi}{\omega} \qquad f = \frac{\omega}{2\pi}$$

$$v = \omega R$$

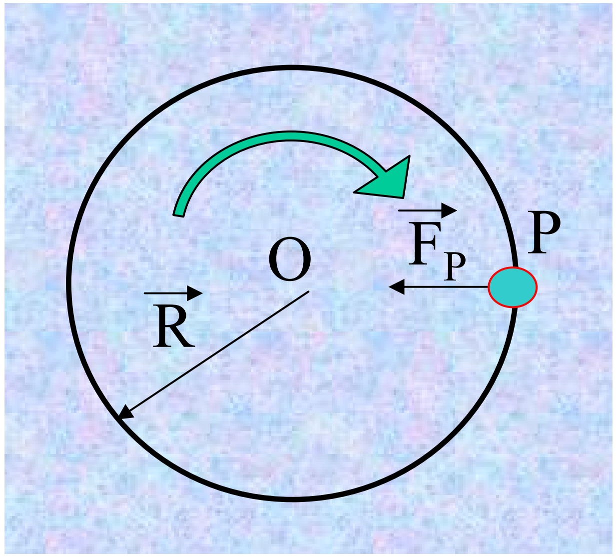
$$a = \frac{v^2}{R} \qquad a = \omega^2 R \qquad \vec{a} = -\frac{v^2}{R}\hat{R} \qquad \vec{a} = -\omega^2 \vec{R}$$

Forza centripeta

$$F = ma = m\frac{\mathrm{v}^2}{R} = m\omega^2 R$$

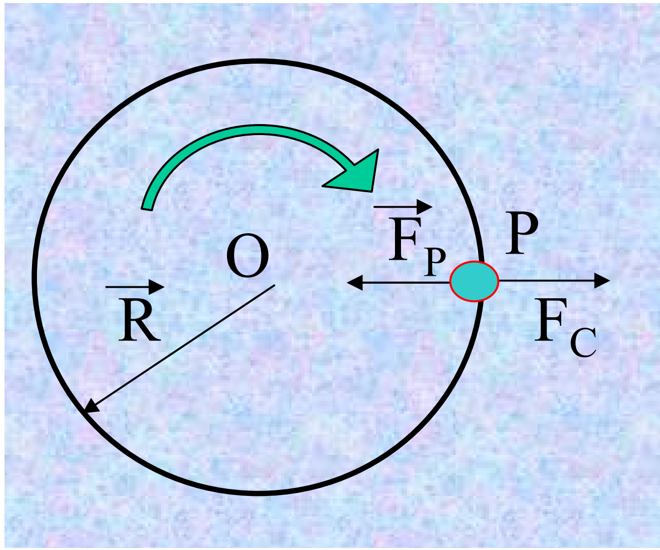
$$\vec{F} = m\vec{a} = -m\frac{\mathrm{v}^2}{R}\hat{R} = -m\omega^2\vec{R}$$

Osservatore fisso



$$\vec{F}_{centripeta} = -m\omega^2 \vec{R}$$

Osservatore solidale con P



$$\vec{F}_{centrifuga} = m\omega^2 \vec{R}$$

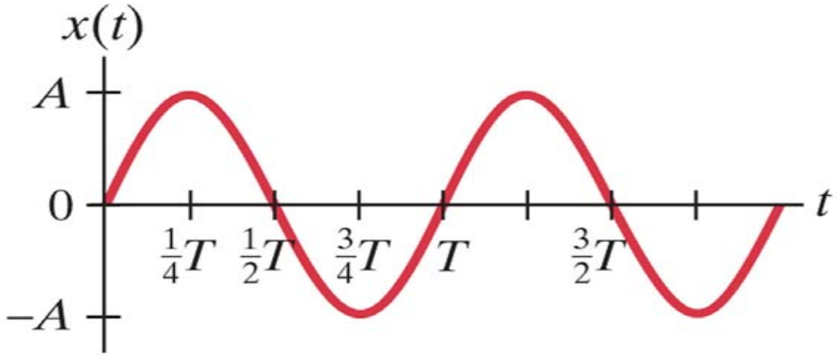
MOTO armonico

Nel moto armonico un corpo percorre avanti e indietro con periodicità una data traiettoria, con una legge del tipo

$$x(t) = A sen(\omega t + \varphi)$$

oppure

$$x(t) = A\cos(\omega t + \varphi)$$



A= ampiezza

T = periodo

$\omega t + \varphi$ fase

$$f = \frac{1}{T}$$ frequenza

φ fase iniziale

ω pulsazione

$$\omega = \frac{2\pi}{T} = 2\pi f$$

$$x(t) = A\ sen(\omega t + \varphi)$$

$$v(t) = \frac{d}{dt}x(t) = \frac{d}{dt}[Asen(\omega t + \varphi)] =$$
$$A[\cos(\omega t + \varphi)]\omega = \omega A\cos(\omega t + \varphi)$$

$$a(t) = \frac{d}{dt}v(t) = \frac{d}{dt}[\omega A\cos(\omega t + \varphi)] =$$
$$\omega A[-sen(\omega t + \varphi)]\omega = -\omega^2 Asen(\omega t + \varphi)$$

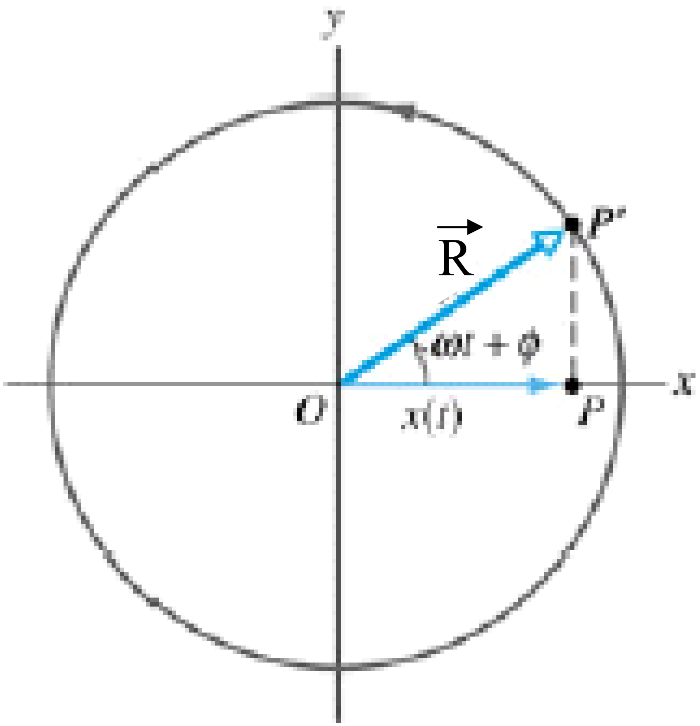
$$a(t) = -\omega^2 x(t)$$

costante positiva

ω = pulsazione

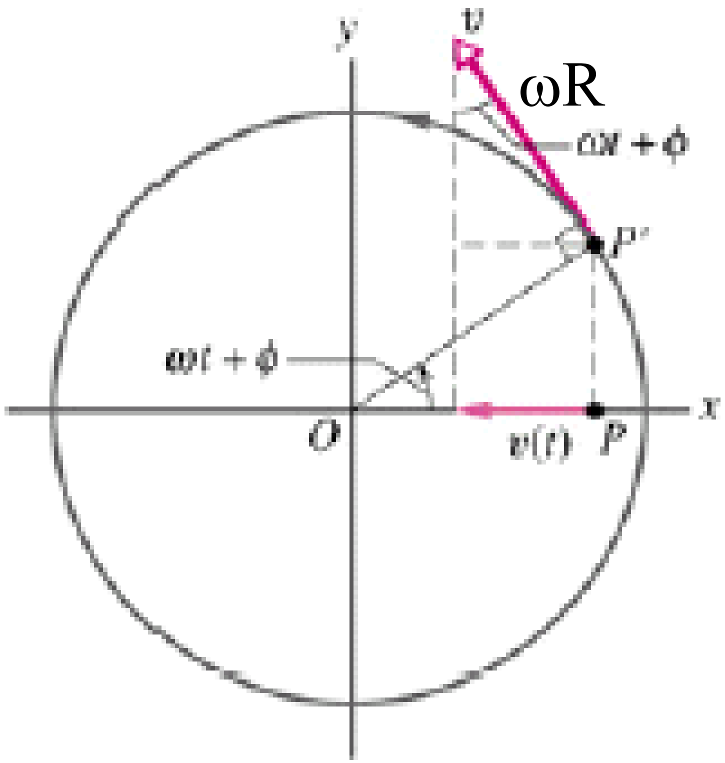
Accelerazione proporzionale allo spostamento ma di segno opposto

Moto armonico come proiezione del moto circolare uniforme



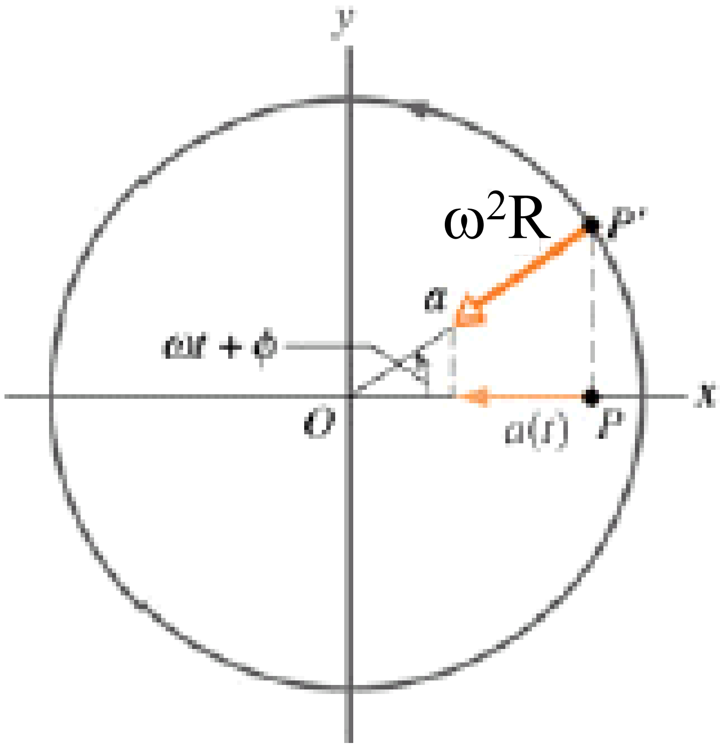
$$x(t) = R\cos(\omega t + \phi)$$

Moto armonico come proiezione del moto circolare uniforme



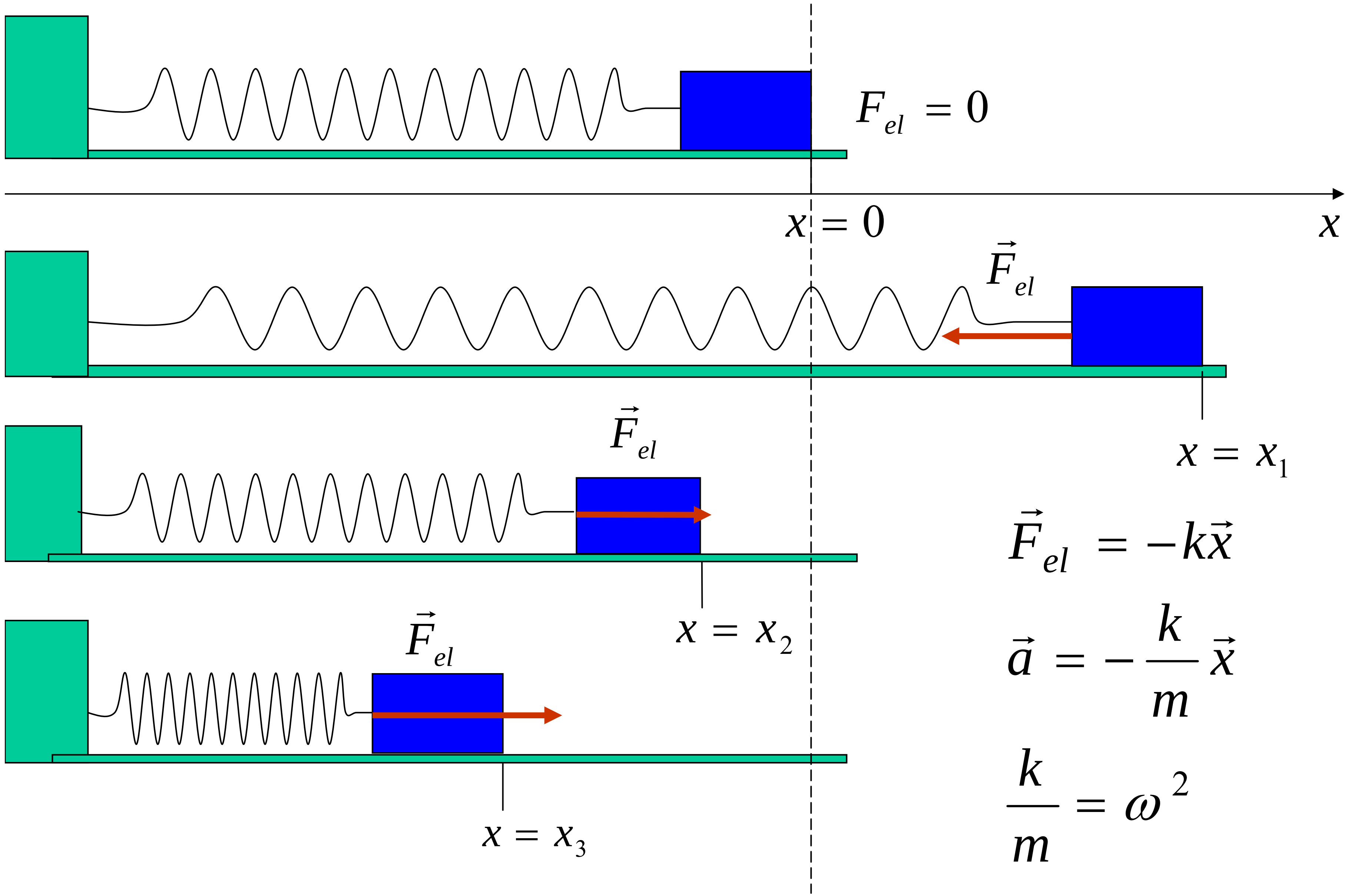
$$v(t) = -\omega R \sin(\omega t + \phi)$$

Moto armonico come proiezione del moto circolare uniforme



$$a(t) = -\omega^2 R \cos(\omega t + \phi)$$

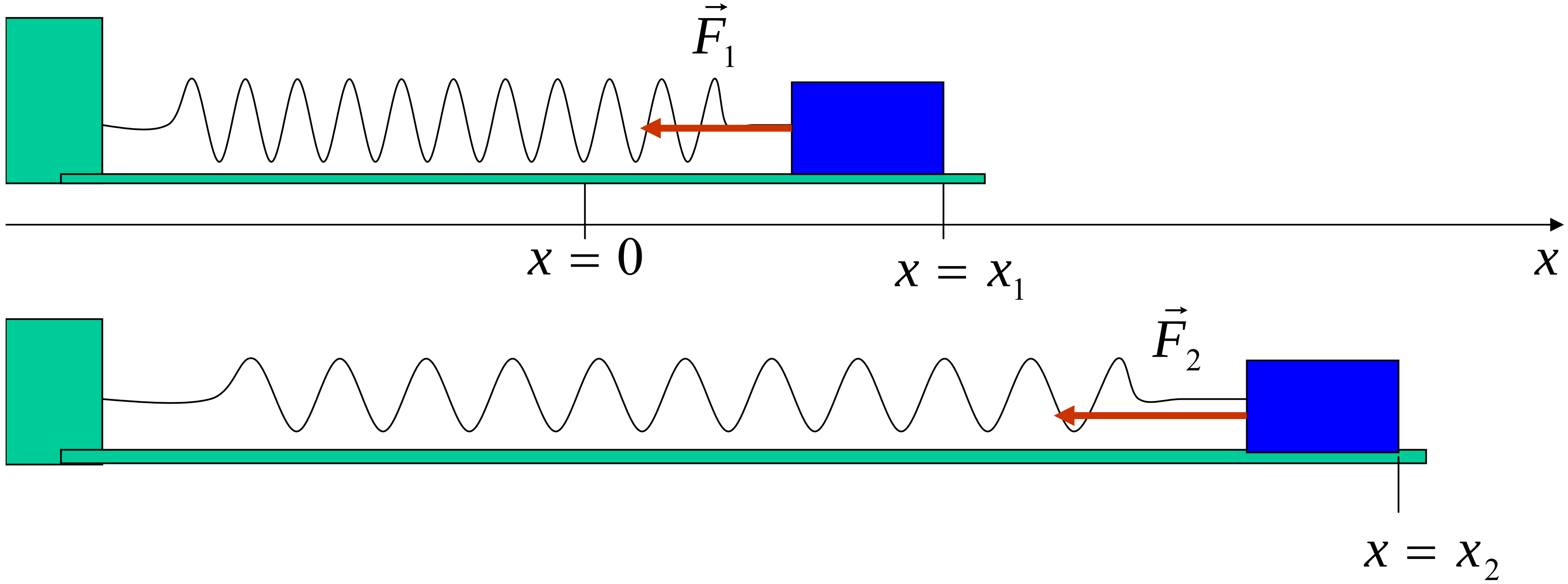
Forza elastica



$$\vec{F}_{el} = -k\vec{x}$$

$$\vec{a} = -\frac{k}{m}\vec{x}$$

$$\frac{k}{m} = \omega^2$$



$$F = -kx$$

$$F = ma$$

$$-kx = ma \Rightarrow a = -\frac{k}{m}x$$

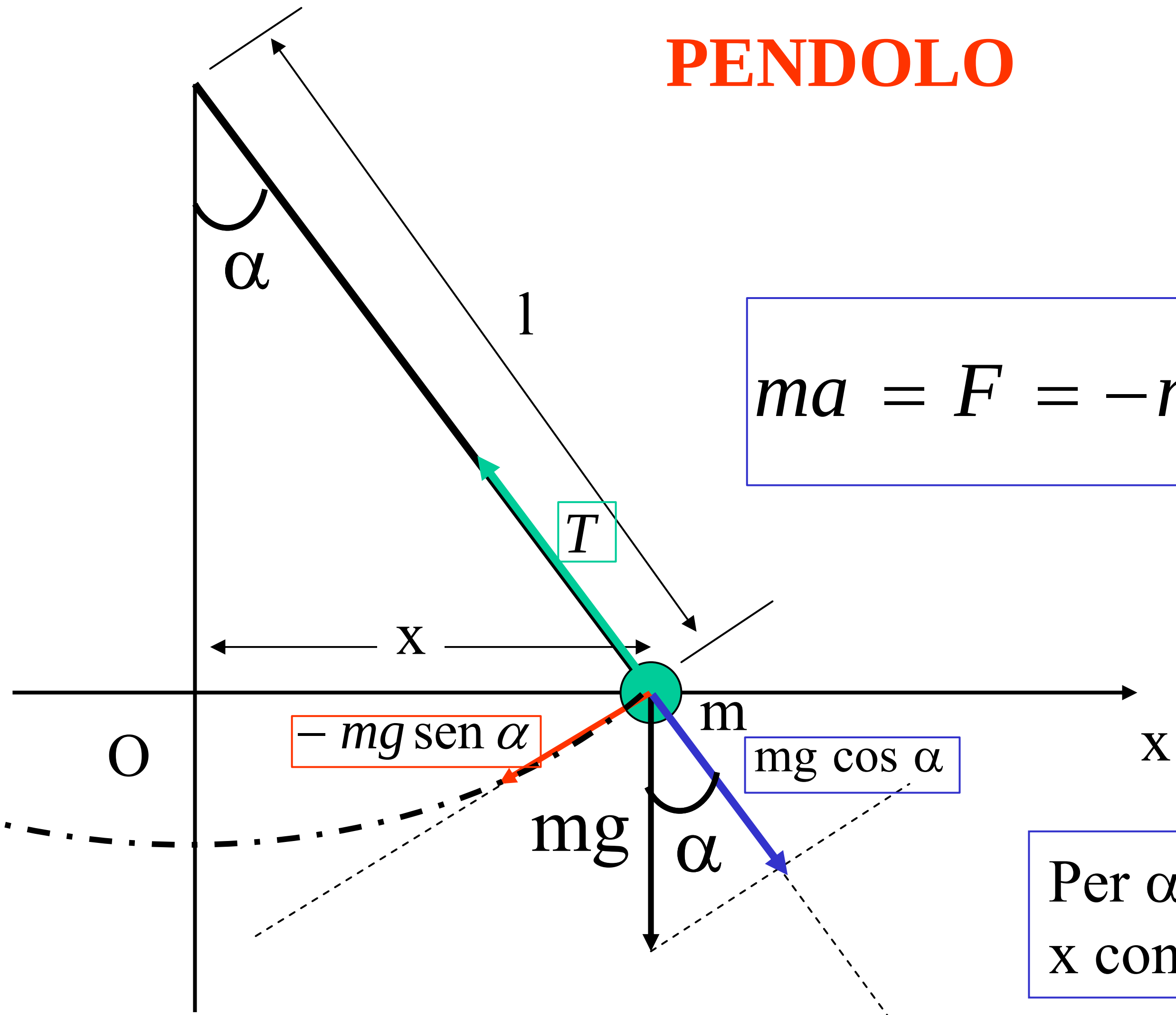
$$\omega^2 = \frac{k}{m}$$

$$x(t) = Asen(\omega t + \varphi) = Asen(\sqrt{\frac{k}{m}}t + \varphi)$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

PENDOLO



$$ma = F = -mg\,\mathrm{sen}\,\alpha = -mg\,\frac{x}{l}$$

$$a = \frac{F}{m} = -g\,\frac{x}{l}$$

Per α piccoli, confondiamo x con la normale al filo

$$a = -\frac{g}{l}x = -\omega^2 x$$

$$\omega = \sqrt{\frac{g}{l}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$$