13/03/19

ES1

$x(t) = -2t^2 + 4t$

$v(t) = \frac{dx}{dt} = -4t + 4$

$a(t) = \frac{dv}{dt} = -4$

ES2

$2(t_f - t_i) = 4\,s$

legge oraria: $y_f = \cancel{y_i}^{\,0} + \cancel{v_{y_i}}^{\,0} t + \frac{1}{2} a_y t^2$

$a_y = g$

$y_f = \frac{1}{2} g (t_f - t_i)^2 = \frac{1}{2} \left(9.81 \frac{m}{s^2}\right) 4\,s^2 = 19.62\,m$

$t_f - t_i = \frac{4\,s}{2} = 2\,s$

ES3

$v_0 = 0\ Km/h$

$v = 100\ Km/h$

$t_1 = 5\,s$

$t_2 = 8\,s$

$t_0 = 0\,s$

(a) $v(t) = \cancel{v_0}^{\,0} + a(t - \cancel{t_0}^{\,0}) \Rightarrow v(t) = at$

$a_1 = \frac{v}{t_1} = \frac{100\ Km/h}{5\,s} = \frac{100 \cdot 10^3\,m}{5\,s \cdot 3.6 \cdot 10^3\,s} = 5.6 \frac{m}{s^2}$

$a_2 = \frac{v}{t_2} = 3.5 \frac{m}{s^2}$

(b) $x(t) = \cancel{x_0}^{\,0} + \cancel{v_0}^{\,0} t + \frac{1}{2} a t^2$

$x(t_1) = \frac{1}{2} a_1 t_1^2 = 0.5 \cdot 5.6 \frac{m}{s^2} \cdot 25\,s^2 = 70\,m$

$x(t_2) = \frac{1}{2} a_2 t_2^2 = 112\,m$

(c) $v_m = \frac{x - x_0}{t - t_0} = \frac{x_0 + v_0 t + \frac{1}{2}at^2 - x_0}{t} = v_0 + \frac{1}{2}at = v_0 + \frac{1}{2}(v - v_0) =$

($t_0 = 0$; $v = v_0 + at$)

$= \frac{v_0 + v}{2} = 50 \frac{Km}{h} = 13.9 \frac{m}{s}$

ES4) $v_A = 100$ Km/h

$v_B = 50$ Km/h

$t_0 = 0$ s

$d_{0\,min}$?

$a = 6 \frac{m}{s^2}$

A: moto unif. acc. $\begin{cases} x_A = x_{A0} + v_A t - \frac{1}{2}at^2 \\ v_A(t) = v_A - at \end{cases}$

B: moto unif. $x_B = x_{B0} + v_B t$

(a) Quando A raggiunge B $v_A(t) = v_B$ e $x_B(t) = x_A(t)$

$d_{0\,min} = x_{B0} - x_{A0} = d$

$x_B(t) = x_A(t)$

$x_{B0} + v_B t = x_{A0} + v_A t - \frac{1}{2}at^2 \Rightarrow d = x_{B0} - x_{A0} = (v_A - v_B)t - \frac{1}{2}at^2 =$

$= (v_A - v_B)\left(\frac{v_A - v_B}{a}\right) - \frac{1}{2}a\left(\frac{v_A - v_B}{a}\right)^2 =$

$= \frac{(v_A - v_B)^2}{2a} = 24\,m$

$v_A(t) = v_B$

$v_A - at = v_B$

$t = \frac{v_A - v_B}{a}$

(b) $d_A = \frac{(v_{Ai} - v_{Af})^2}{2a} = \frac{(v_{Ai})^2}{2a} = \left[100 \cdot \frac{10^3 m}{3.6 \cdot 10^3 s}\right]^2 \cdot \frac{1}{8}\frac{s^2}{m} = 96.5\,m$

(c) $x_A = x_{A0} + v_A t$ A: prima moto unif. poi unif. acc

$x_B = x_{B0} + v_B t$ B: sempre moto unif

$\Delta x_A = v_A t = 27.8\ m$

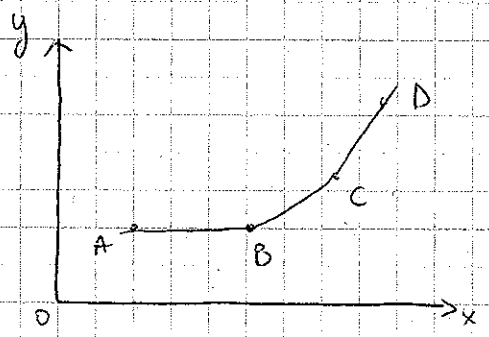
$100 \cdot \frac{10^3\ m}{3.6 \cdot 10^3\ s} \cdot 1\ s$

Δt = tempo di reazione

$\Delta x_B = v_B t = 13.9\ m$

$d_{min} = (v_A - v_B)\Delta t + \frac{(v_A - v_B)^2}{2a} = 13.9\ m + 24\ m \sim 38\ m$

Es 5



(a) $x_B = x_A + v_A t = 3\ m + 5\ \frac{m}{s} \cdot 2\ s = 13\ m$

B (13 m, 2 m)

(b) $v_{Bx} = 5\ m/s$ (moto unif lungo x)

$v_{By} = 0\ m/s$

(c) $x_C = x_B + v_A t = 13\ m + 5\ \frac{m}{s} \cdot 4\ s = 33\ m$

$y_C = y_B + v_{y_B} t_{BC} + \frac{1}{2} a_{BC} t_{BC}^2 =$

$= 2\ m + \frac{1}{2}\ 4\ m/s^2\ 16\ s^2 = 34\ m$

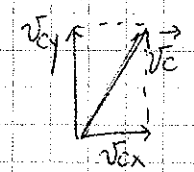
C (33 m, 34 m)

(d) $v_{Cx} = 5\ m/s$

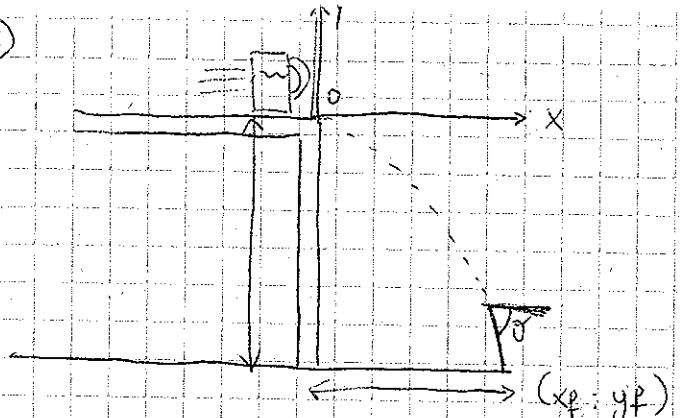
$v_{Cy} = \underset{0}{v_{By}} + a_{BC} t_{BC} = 4\ m/s^2 \cdot 4\ s = 16\ m/s$

$|\vec{v_C}| = \sqrt{v_{Cx}^2 + v_{Cy}^2} = \sqrt{(5^2 + 16^2)\ m^2/s^2} = \sqrt{25 + 256}\ m/s = 16.8\ m/s$

$tg\theta = \frac{v_{Cy}}{v_{Cx}} = \frac{16}{5} = 3.2 \qquad \theta = arctg(3.2) = 72.6°$



ES6



$x_f = 1.60\ m$

$a_x = 0$

$y_f = -0.860\ m$

$v_{yi} = 0$

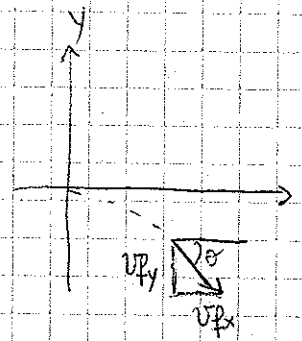
$a_y = -9.81\ \frac{m}{s^2}$

(a) $y_f = v_{yi} t + \frac{1}{2} a_y t^2$

$-0.860\ m = 0 + \frac{1}{2}(-9.8\ m/s^2) t^2 \Rightarrow t = 0.419\ s$

$\Rightarrow v_{xf} = \frac{x_f}{t} = \frac{1.60\ m}{0.419\ s} = 3.34\ \frac{m}{s}$

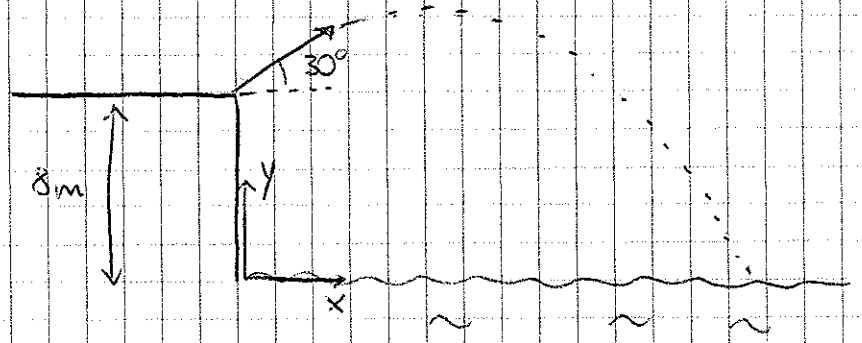
(b)



$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{yx}}\right) = \tan^{-1}\left(\frac{-4.11\ m/s}{3.34\ m/s}\right) = -50.9°$

20/3/19

ES1



(a) leggi orarie:

$$x(t) = 15\cos(30^\circ)\,t$$

$$y(t) = 8 + 15\sin(30^\circ)\,t - 4.905\cdot t^2$$

t_v = tempo di volo

$$y(t_v) = 0 = 8 + 15\sin 30^\circ\, t_v - 4.905\, t_v^2 \Rightarrow t_v \simeq 2.25\ s$$

(b) $x(t_v) = 15\cdot\cos 30^\circ \cdot t_v \simeq 29.3\ m$

(c) Sia t_s = tempo di salita.

$$t_s = \frac{15\sin 30^\circ}{9.81}$$

$$y(t_s) = 8 + 15\sin 30^\circ\, t_s - 4.905\cdot t_s^2 \simeq 10.9\ m$$

(d) $x(t_s) = 15\cos 30^\circ\, t_s \simeq 9.93\ m$

(e) $v_x(t_s) = v_{0x} = 15\cos 30^\circ \simeq 13\ \frac{m}{s}$

(f) $v_x(t_v) = v_{0x} = 15\cos 30^\circ \simeq 13\ m/s$

$$v_y(t_v) = v_{0y} - 9.81\, t_v = 15\sin 30^\circ - 9.81\, t_v \simeq -14.6\ m/s$$

$$v = \sqrt{v_x^2 + v_y^2} \simeq 19.5\ m/s$$

(g) $$\alpha = \tan^{-1}\left(\frac{v_y}{v_x}\right) \simeq 48.3^\circ$$

ES2 Il periodo è $T = \frac{18\,s}{5+\frac{2}{3}} \simeq 3.18\,s$.

La velocità angolare è $\omega = \frac{2\pi}{T} \simeq 1.98\ \frac{rad}{s}$

(a) $a_c = \omega^2 R \Rightarrow R = \frac{a_c}{\omega^2} \simeq 1.79\,m$

(b) $v = \omega R \Rightarrow v \simeq 3.5\ \frac{m}{s}$

(c) $f = \frac{1}{T} = 0.31\ Hz$

ES3 Lancetta delle ore: $T_{ore} = 12 \cdot 3600\,s = 43200\,s \Rightarrow \omega_{ore} = \frac{2\pi}{T_{ore}} \simeq 0{,}15 \cdot 10^{-3}\ rad/s$

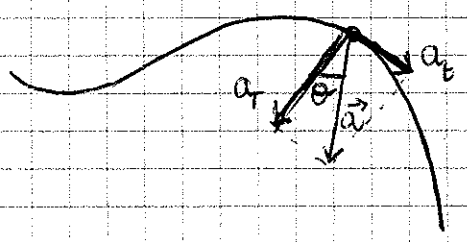
Lancetta dei minuti: $T_{min} = 60 \cdot 60 = 3600\,s \Rightarrow \omega_{min} = \frac{2\pi}{T_{min}} \simeq 0.17 \cdot 10^{-2}\ rad/s$

Lancetta dei secondi: $T_{sec} = 60\,s \Rightarrow \omega_{sec} = \frac{2\pi}{T_{sec}} \simeq 0.105\ \frac{rad}{s}$

ES4 Moto circolare non uniforme

$a = a_r + a_t$ ← componente tangenziale

↑ componente radiale



$50\ Km/h = 50\ \frac{10^3\,m}{3.6 \cdot 10^3\,s} = 13.89\ \frac{m}{s}$

$90\ Km/h = 25\ \frac{m}{s}$

Quando $v = 13.89\ m/s$

$$a_r = -\frac{v^2}{r} = -\frac{(13.89\ m/s)^2}{150\ m} = -1.29\ m/s^2$$

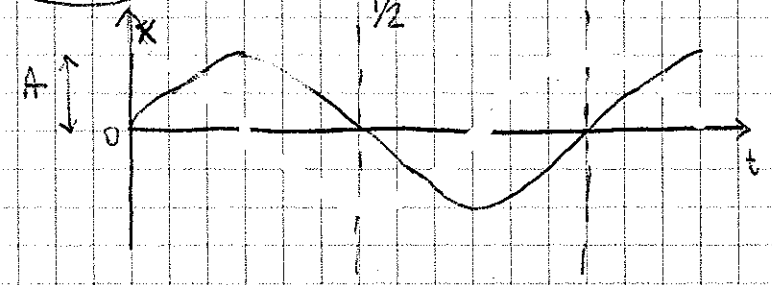
$$a_t = \frac{\Delta v}{\Delta t} = \frac{13.89\ m/s - 25.0\ m/s}{15\ s} = -0.741\ \frac{m}{s^2}$$

↑ variazione del modulo della velocità tangenziale

$$\Rightarrow a = \sqrt{a_r^2 + a_t^2} = 1.48\ \frac{m}{s^2}$$

$$\theta = \tan^{-1}\left(\frac{a_t}{a_r}\right) = \tan^{-1}\left(\frac{0.741}{1.29}\right) = 29.9°$$

ES 5



(a) a $t=0$, $x=0$

$x(t) = A\sin(\omega t + \phi)$ in generale

ϕ = angolo che indica a quale punto della traiettoria il punto materiale ha iniziato il suo moto armonico

$A = 2\ cm$

$\phi = 0$

$\omega = 2\pi f = 2\pi(1.5) = 3\pi$

$x(t) = (2\ cm)\ sen(3\pi t)$

(b) $x = A\,\text{sen}(\omega t)$

$v = A\omega\cos(\omega t)$

$a = -A\omega^2\,\text{sen}(\omega t)$

$v = 2(3\pi)\cos(3\pi t)$

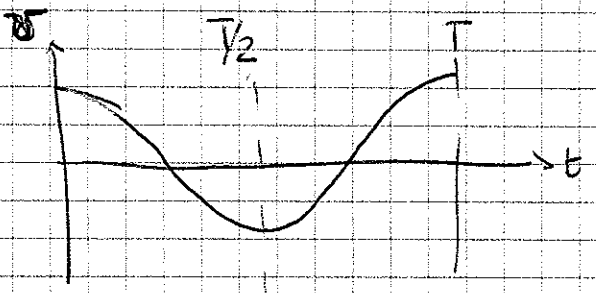
↳ cos max quando vale 1

$v_{max} = 6\pi\ \frac{cm}{s}$

La particella ha questa velocità quando $\cos(3\pi t) = 1$

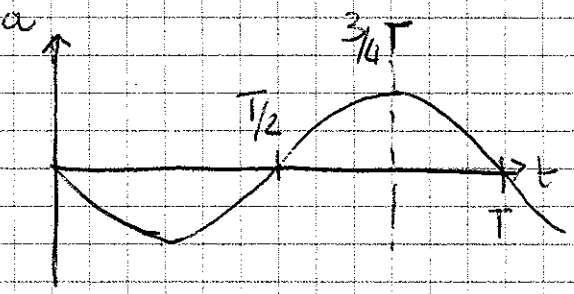
$\Rightarrow$ in $t = 0$

E successivamente per $t = \frac{T}{2}$: $\left(\cos\left(3\pi\frac{T}{2}\right)\right) = -1$



(c) $a = -2(3\pi)^2\,\text{sen}(3\pi t)$

$a_{max} = A\omega^2 = 2(3\pi)^2 = 178\ \frac{cm}{s^2}$



$t_{max} = \frac{3}{4}T = 0.5s$