Geometry of Schemes

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Exercise Sheet 1

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Exercise 1.1 Let $\mathcal{B}$ be a base of open subsets on a topological space X (with this we mean that $\mathcal{B}$ is closed under finite intersections). Let $\mathcal{F}$ and $\mathcal{G}$ be two sheaves (of abelian groups) on X. Suppose that, for any $U \in \mathcal{B}$ there exists a homomorphism $\alpha(U): \mathcal{F}(U) \to \mathcal{G}(U)$ which is compatible with restrictions. Show that this extends in a unique way to a morphism of sheaves $\alpha: \mathcal{F} \to \mathcal{G}$.

Exercise 1.2 Let X be a scheme and let $f \in \mathcal{O}_X(X)$. Show that the association $U \mapsto f|_U \mathcal{O}_X(U)$, for every open subset $U \subseteq X$, defines a sheaf of ideals on X. Show that the support of this sheaf is closed.

Exercise 1.3 Let A be a graded ring and let Z be a reduced closed subscheme of $\mathrm{Proj}(A)$. Show that there exists a homogeneous ideal I of A such that $Y \cong \mathrm{Proj}(A/I)$.

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