Geometry of Schemes

Academic Year 2016/2017

Exercise Sheet 2

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Exercise 2.1 Let A be a ring. We say that $e \in A$ is *idempotent* is $e \neq 0$ and $e^2 = e$. Let X be a scheme. Show that the following are equivalent:

(a) X is connected;

(b) $\mathcal{O}_X(X)$ has no other idempotent elements than 1;

(c) $\mathrm{Spec}(\mathcal{O}_X(X))$ is connected.

Exercise 2.2 A morphism of schemes $f : X \to Y$ is *finite* is there is an open affine cover $\{V_i = \mathrm{Spec}(B_i) : i \in I\}$ of Y such that, for each $i \in I$, $f^{-1}(V_i) = \mathrm{Spec}(A_i)$ is affine and A_i is a finitely generated B_i-module. Show that a finite morphism is proper.

Exercise 2.3 Let $f : X \to Y$ be a separated morphism. Prove that any section of f is a closed immersion.