

# Geometry of Schemes

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Exercise Sheet 2

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**Exercise 2.1** Let  $A$  be a ring. We say that  $e \in A$  is *idempotent* if  $e \neq 0$  and  $e^2 = e$ . Let  $X$  be a scheme. Show that the following are equivalent:

- (a)  $X$  is connected;
- (b)  $\mathcal{O}_X(X)$  has no other idempotent elements than 1;
- (c)  $\text{Spec}(\mathcal{O}_X(X))$  is connected.

**Exercise 2.2** A morphism of schemes  $f : X \rightarrow Y$  is *finite* if there is an open affine cover  $\{V_i = \text{Spec}(B_i) : i \in I\}$  of  $Y$  such that, for each  $i \in I$ ,  $f^{-1}(V_i) = \text{Spec}(A_i)$  is affine and  $A_i$  is a finitely generated  $B_i$ -module. Show that a finite morphism is proper.

**Exercise 2.3** Let  $f : X \rightarrow Y$  be a separated morphism. Prove that any section of  $f$  is a closed immersion.

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